**Resolution Refutation examples**

**Example 1:**

Consider the following axioms:

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (Conclusion) If John is a light sleeper, then John does not have any mice.

The conclusion can be proved using Resolution as shown below. The first step is to write each axiom as a well-formed formula in first-order predicate calculus. The clauses written for the above axioms are shown below, using LS(x) for `light sleeper'.

1. *∀ x (HOUND(x) → HOWL(x))*
2. *∀ x ∀ y (HAVE (x,y) ∧ CAT (y) → ¬ ∃ z (HAVE(x,z) ∧ MOUSE (z)))*
3. *∀ x (LS(x) → ¬ ∃ y (HAVE (x,y) ∧ HOWL(y)))*
4. *∃ x (HAVE (John,x) ∧ (CAT(x) ∨ HOUND(x)))*
5. *LS(John) → ¬ ∃ z (HAVE(John,z) ∧ MOUSE(z))*

The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

1. *∀ x (HOUND(x) → HOWL(x))*

*¬ HOUND(x) ∨ HOWL(x)*

1. *∀ x ∀ y (HAVE (x,y) ∧ CAT (y) &rarr ¬ ∃ z (HAVE(x,z) ∧ MOUSE (z)))*

*∀ x ∀ y (HAVE (x,y) ∧ CAT (y) &rarr ∀ z ¬ (HAVE(x,z) ∧ MOUSE (z)))*

*∀ x ∀ y ∀ z (¬ (HAVE (x,y) ∧ CAT (y)) ∨ ¬ (HAVE(x,z) ∧ MOUSE (z)))*

*¬ HAVE(x,y) ∨ ¬ CAT(y) ∨ ¬ HAVE(x,z) ∨ ¬ MOUSE(z)*

1. *∀ x (LS(x) &rarr ¬ ∃ y (HAVE (x,y) ∧ HOWL(y)))*

*∀ x (LS(x) → ∀ y ¬ (HAVE (x,y) ∧ HOWL(y)))*

*∀ x ∀ y (LS(x) → ¬ HAVE(x,y) ∨ ¬ HOWL(y))*

*∀ x ∀ y (¬ LS(x) ∨ ¬ HAVE(x,y) ∨ ¬ HOWL(y))*

*¬ LS(x) ∨ ¬ HAVE(x,y) ∨ ¬ HOWL(y)*

1. *∃ x (HAVE (John,x) ∧ (CAT(x) ∨ HOUND(x)))*

*HAVE(John,a) ∧ (CAT(a) ∨ HOUND(a))*

1. *¬ [LS(John) → ¬ ∃ z (HAVE(John,z) ∧ MOUSE(z))] (negated conclusion)*

*¬ [¬ LS (John) ∨ ¬ ∃ z (HAVE (John, z) ∧ MOUSE(z))]*

*LS(John) ∧ ∃ z (HAVE(John, z) ∧ MOUSE(z)))*

*LS(John) ∧ HAVE(John,b) ∧ MOUSE(b)*

The set of CNF clauses for this problem is thus as follows:

1. *¬ HOUND(x) ∨ HOWL(x)*
2. *¬ HAVE(x,y) ∨ ¬ CAT(y) ∨ ¬ HAVE(x,z) ∨ ¬ MOUSE(z)*
3. *¬ LS(x) ∨ ¬ HAVE(x,y) ∨ ¬ HOWL(y)*
   1. *HAVE(John,a)*
   2. *CAT(a) ∨ HOUND(a)*
   3. *LS(John)*
   4. *HAVE(John,b)*
   5. *MOUSE(b)*

Now we proceed to prove the conclusion by resolution using the above clauses. Each result clause is numbered; the numbers of its parent clauses are shown to its left.

|  |  |  |
| --- | --- | --- |
| [1.,4.(b):] | 6. | *CAT(a) ∨ HOWL(a)* |
| [2,5.(c):] | 7. | *¬ HAVE(x,y) ∨ ¬ CAT(y) ∨ ¬ HAVE(x,b)* |
| [7,5.(b):] | 8. | *¬ HAVE(John,y) ∨ ¬ CAT(y)* |
| [6,8:] | 9. | *¬ HAVE(John,a) ∨ HOWL(a)* |
| [4.(a),9:] | 10. | *HOWL(a)* |
| [3,10:] | 11. | *¬ LS(x) ∨ ¬ HAVE(x,a)* |
| [4.(a),11:] | 12. | *¬ LS(John)* |
| [5.(a),12:] | 13. | *□* |
|  |  |  |

**Example 2:**

See the link below

[Resolution in First-order logic - Javatpoint](https://www.javatpoint.com/ai-resolution-in-first-order-logic)